

Negative Echo in the Density Evolution of Ultracold Fermionic Gases

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We predict a nonequilibrium critical phenomenon in the space-time density evolution of a fermionic gas above the temperature of transition into the superfluid phase. On the BCS side of the BEC-BCS crossover, the evolution of a localized density disturbance exhibits a negative echo at the point of the initial inhomogeneity. Approaching the BEC side, this effect competes with the slow spreading of the density of bosonic molecules. However, even here the echo dominates for large enough times. This effect may be used as an experimental tool to locate the position of the transition.

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Superconducting [or superfluid] fluctuations (SF) have a profound effect on the DC transport and the thermodynamic properties of superconductors, leading to the famous Aslamazov-Larkin and Maki-Thompson (MT) corrections to the conductivity [1, 2]. Valuable information about the role of SF in the emerging correlations of the many-body wave functions is provided by the real time dynamics. Unfortunately, the latter can hardly be probed in solid state experiments.

On the side of atomic physics, recent studies of fermionic ultracold atomic gases (UCAG) with interparticle interaction that can be fine-tuned by exploiting Fano-Feshbach resonances (FFr)[3–6], have revealed pronounced effects and found clear signatures of superfluidity [7–14] in the regime of the so-called BEC-BCS crossover [15]. In contrast to solid state systems, where it is hard to observe a dynamical response to external perturbations, in trapped UCAG it is possible to probe the actual evolution of a disturbance in real time and space. This offers, among other things, the exciting possibility of discovering novel manifestations of the peculiar physics of SF.

In this Letter, we analyze the nonequilibrium space and time evolution of density in the BEC-BCS crossover, and find that even above the critical temperature SF qualitatively affect the dynamics. The quantum kinetic equations (QKE) we derive here, allow us to analyze dynamical effects in the critical region.

Suppose that originally (at $t = 0$) the density is depleted in a small region inside the system. The time evolution of this density dip, illustrated by Fig. 1, turns out to be determined by how close the temperature T is to its critical value T_c , and by the position of the system along the crossover. In the BCS limit, where T_c is low, we predict the development of a sharp *peak* in the density of fermionic atoms at the origin (see Fig. 1a). The shape of the peak is critical: as $T \rightarrow T_c^+$ the width shrinks as $\tau^{1/2}$ ($\tau \equiv (T - T_c)/T_c$). As the resonance detuning is further lowered, T_c increases and corrections to the pure BCS behavior become important: the bosonic molecules depleted initially produce a dip that dominates the fermionic peak for small times and

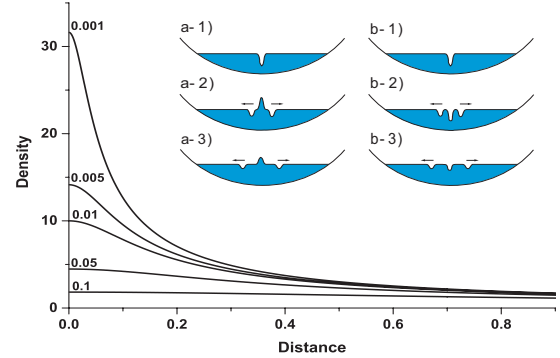


FIG. 1: (Color online) Density profile $\delta\rho_F(\mathbf{x}, t)$, Eq. (16), in arbitrary units as a function of the scaled distance from the origin $\sqrt{\pi b/\gamma(E_1)}mr/t$, for values of τ shown on the curves. The inset sketches the total density evolution in regions III-IV (a) and II (b). However, due to the exponential decay of the bosonic dip, even in region II one always ends up with an algebraically decaying fermionic peak (a-3) for large times.

τ 's (see Fig. 1b and region II of Fig. 2). Eventually, however, the bosons decay and the fermionic peak emerges. We will find that tuning the system towards the bosonic side of the crossover reduces the interval of τ 's where the fermionic density peak is observable. Eventually, in the BEC regime only the bosonic dip remains. To measure these effects, we propose the experiment shown in Fig. 1a with fermionic UCAG close to the superfluid transition, following the experimental procedure introduced by [16], e.g. by applying the repulsive optical dipole force of a strongly focused off-resonant laser beam. We stress that the negative echo effect is not limited to the far BCS limit and is observable in the region that is accessible to current experiments.

The profile of the density peak formed by fermions in the BCS side of the crossover, is plotted in Fig. 1. The microscopic mechanism at the root of this phenomenon is the Andreev reflection of fermionic atoms off the fluctuations of the superfluid order parameter. The wavelength of such fluctuations diverges at $T \rightarrow T_c^+$. It is well known [17] that an electronic excitation, with sub-gap energy, incident on a superconductor/normal metal interface from

the normal side, will be reflected back as a hole, almost opposite to its original direction of motion, creating a Cooper pair in the superconductor. Although the superconducting gap vanishes above the transition point, close to T_c there appear critically large fluctuating superfluid islands which transform incident nonequilibrium particle-like excitations into holes, and vice versa. Thus the time evolution shown in Fig. 1a can be understood as follows. The initial density depletion can be thought of as an excess of holes, which at $t = 0$ start to move radially outwards from the origin, while the fluctuating condensate reflects them back towards the origin, in the form of atoms rather than holes. Possible deviation of this reflection from exact backscattering is determined by the typical size of a condensate island, which diverges as $\tau^{-1/2}$. As a result fermions accumulate at the origin forming a critically sharp density peak, the width of which scales as $\tau^{1/2}$ (see Eq. (16)).

On the other hand, the density dip of Fig. 1b, is due to the slow propagation of the bosonic component of the initial nonequilibrium disturbance away from the origin (see Eq. (17)).

The fermionic atoms interacting via a FFr are described by [18–20]

$$\begin{aligned} \hat{H} = & \int d\mathbf{r} \left[\sum_{\sigma} \hat{a}_{\sigma}^{\dagger} \epsilon_a(\mathbf{r}, -i\hbar\nabla) \hat{a}_{\sigma} + \hat{b}^{\dagger} \epsilon_b(\mathbf{r}, -i\hbar\nabla) \hat{b} \right] \\ & + \frac{g}{2} \int d\mathbf{r} \sum_{\sigma_1 \sigma_2} \left[\sigma_{\sigma_1 \sigma_2}^y \hat{b} \hat{a}_{\sigma_1}^{\dagger} \hat{a}_{\sigma_2}^{\dagger} + h.c. \right], \quad (1a) \\ \epsilon_a = & \mathbf{p}^2/(2m) + U_a(\mathbf{r}, t); \quad \epsilon_b = \mathbf{p}^2/(4m) + U_b(\mathbf{r}, t), \quad (1b) \end{aligned}$$

where $\hat{a}_{\sigma}^{\dagger}(\mathbf{r})$, $[\hat{a}_{\sigma}(\mathbf{r})]$ create [annihilate] fermionic atoms with mass m and spin $\sigma = \pm 1/2$, while $\hat{b}^{\dagger}(\mathbf{r})$, $[\hat{b}(\mathbf{r})]$ create [annihilate] the bosonic molecules. $U_{a(b)}$ is the sum of the confining and perturbing potentials acting on an atom (molecule).

Given the energy of relative motion of two atoms, ϵ , their s -channel scattering amplitude can be written as

$$f_0(\epsilon) = \hbar(m\epsilon)^{-1/2} \gamma(\epsilon) [\epsilon_0 - \epsilon - i\gamma(\epsilon)]^{-1}, \quad (2)$$

with the resonance width $\gamma(\epsilon) = \frac{mg^2\sqrt{m\epsilon}}{4\pi\hbar^3}$, and physical position ϵ_0 . Physical results expressed in terms of ϵ_0 and γ are well defined [21].

QKE for the normal phase are derived (see *e.g.* Ref. [22]) by finding a semiclassical approximation for the time ordered Green functions. We combine the latter into 2×2 matrices,

$$\hat{\mathcal{G}} = \begin{pmatrix} \hat{\mathcal{G}}^K & \hat{\mathcal{G}}^R \\ \hat{\mathcal{G}}^A & 0 \end{pmatrix}; \quad \hat{\mathcal{D}} = \begin{pmatrix} D^K & D^R \\ D^A & 0 \end{pmatrix}, \quad (3)$$

and $\hat{\mathcal{G}}^{K/R/A}$ are 2×2 matrices in the spin space.

We limit ourselves to the condition of narrow resonance, (see Ref. [23] for the experimental realization)

$$\gamma(E) \ll E, \quad E = \max(T, \epsilon_F), \quad 2m\epsilon_F \equiv [3\pi^2\hbar^3\rho_a]^{\frac{2}{3}}, \quad (4)$$

where ρ_a is the atomic density in the absence of the resonance, and T is the temperature [24]. For $\tau \gg Gi$, where $Gi \ll 1$ is the Ginzburg parameter [25], condition (4) justifies the one loop approximation

$$\hat{\mathcal{G}} = \hat{\mathcal{G}}_0 + \text{diagram with loop} \quad (5a)$$

$$\hat{\mathcal{D}} = \hat{\mathcal{D}}_0 + \text{diagram with loop} \quad (5b)$$

$$\text{vertex} = \frac{-ig\sigma_{\sigma_1\sigma_2}^y d_{klm}}{\sqrt{2}}, \quad (5c)$$

for the derivation of the QKE. Here the structure of the vertices in the Keldysh space (3) is defined by $d_{klm} = \delta_{k2}\delta_{lm} + \delta_{l2}\delta_{km} + \delta_{m2}\delta_{kl} - 2\delta_{l2}\delta_{k2}\delta_{m2}$. As usual [22], the derivation of the QKE requires obtaining the equation for the Wigner transforms for the Green function ($\mathbf{r}_{1,2} \equiv \mathbf{r} \pm \delta\mathbf{r}/2$, $t_{1,2} \equiv t \pm \delta t/2$):

$$\left[\begin{matrix} \hat{\mathcal{G}}(\mathbf{r}_1, t_1; \mathbf{r}_2, t_2) \\ \hat{\mathcal{D}}(\mathbf{r}_1, t_1; \mathbf{r}_2, t_2) \end{matrix} \right] = \int \frac{d\mathbf{p} d\epsilon}{(2\pi\hbar)^4} e^{\frac{i(\mathbf{p}\delta\mathbf{r} - \epsilon\delta t)}{\hbar}} \left[\begin{matrix} \hat{\mathcal{G}}(\epsilon, \mathbf{X}) \\ \hat{\mathcal{D}}(\epsilon, \mathbf{X}) \end{matrix} \right], \quad (6)$$

and keeping leading terms in \hbar expansion. [Hereinafter, we use the notation $\mathbf{X} \equiv (\mathbf{p}, \mathbf{x})$, $\mathbf{x} \equiv (\mathbf{r}, t)$.]

We parametrize the Keldysh components of matrices (3) in terms of the distribution functions $\hat{n}(\mathbf{X})$, $N(\mathbf{X})$:

$$\begin{aligned} \hat{\mathcal{G}}^K(\epsilon, \mathbf{X}) &= -i\pi \left[\delta(\epsilon - \hat{\mathcal{E}}(\mathbf{X})) , 1 - 2\hat{n}(\mathbf{X}) \right]_+, \quad (7) \\ D^K(\epsilon, \mathbf{X}) &= -2i\pi\delta(\epsilon - \Omega(\mathbf{X})) [2N(\mathbf{X}) + 1], \end{aligned}$$

with $\hat{n}(\mathbf{X})$ a 2×2 matrix in the spin space. Hereinafter, $[\cdot, \cdot]_+$ means anticommutator. Using Eqs. (5)–(7), we derive the QKE (details will be published elsewhere):

$$\partial_t \hat{n} + \frac{i}{\hbar} [\hat{\mathcal{E}}, \hat{n}] + \left\{ \hat{\mathcal{E}}, \hat{n} \right\}_P = \int \frac{d\mathbf{p}_1}{(2\pi\hbar)^3} \hat{S}t(\mathbf{p}, \mathbf{p}_1; \mathbf{x}); \quad (8a)$$

$$\partial_t N + \{\Omega, N\}_P = - \int \frac{d\mathbf{p}_1}{2(2\pi\hbar)^3} \text{Tr} \hat{S}t(\mathbf{p} - \mathbf{p}_1, \mathbf{p}_1; \mathbf{x}), \quad (8b)$$

where $[\cdot, \cdot]$ stand for the commutator and the Poisson brackets for arbitrary matrices \hat{A}, \hat{B} are defined as $2\{\hat{A}, \hat{B}\}_P = [\nabla_{\mathbf{p}}\hat{A}, \nabla_{\mathbf{r}}\hat{B}]_+ - [\nabla_{\mathbf{r}}\hat{A}, \nabla_{\mathbf{p}}\hat{B}]_+$. Equation (8b) is valid when the bosonic spectral width is smaller than temperature, and thus is not valid in the far BCS region (region IV of Fig. 2). However, it can be shown that in this region it suffices to use the equilibrium bosonic distribution (see Eq. (13)) in Eq. (8a), and Eq. (8b) is not needed.

In Eq. (8b), the energies of the fermionic and bosonic excitations, $\hat{\mathcal{E}}$ and Ω , depend on the momentum and the coordinate through the bare energies (1b) and also on the distribution functions of the excitations themselves. Due to a possible non-equilibrium spin population, the

spectrum $\hat{\mathcal{E}}$ may possess a spin space structure:

$$\hat{\mathcal{E}}(\mathbf{X}; \{\hat{n}, N\}) = \frac{\mathbf{p}^2}{2m} + U_a(\mathbf{r}, t) - \frac{g^2}{2} \int \frac{d\mathbf{p}_1}{(2\pi\hbar)^3} \left[\hat{n}(\mathbf{p}_1, \mathbf{x}) + N(\mathbf{p} + \mathbf{p}_1, \mathbf{x}), \frac{1}{\hat{\Delta}(\mathbf{p}, \mathbf{p}_1; \mathbf{x})} \right]_+ \quad (9a)$$

where notation $\hat{\Delta} \equiv \Omega(\mathbf{p}_1 + \mathbf{p}_2, \mathbf{x}) - \hat{\mathcal{E}}(\mathbf{p}_1, \mathbf{x}) - \hat{\mathcal{E}}(\mathbf{p}_2, \mathbf{x})$ is introduced and overline signifies the time reversal operation in the spin space: $\overline{\hat{A}} \equiv \hat{\sigma}_y \hat{A}^T \hat{\sigma}_y$. The spectrum of the bosonic excitations acquires a shift

$$\Omega(\mathbf{X}; \{\hat{n}\}) = \frac{\mathbf{p}^2}{4m} + U_b(\mathbf{r}, t) + g^2 \int \frac{d\mathbf{p}_1}{(2\pi\hbar)^3} \left[\text{Tr} \frac{1/2 - \hat{n}(\mathbf{p}_1, \mathbf{x})}{\hat{\Delta}(\mathbf{p} - \mathbf{p}_1, \mathbf{p}_1, \mathbf{x})} + \frac{m}{\mathbf{p}_1^2} \right], \quad (9b)$$

which for a sharp fermionic distribution function, diverges logarithmically as in conventional BCS.

Real collision processes defining the irreversible evolution in Eq. (8b) are described by (we omitted \mathbf{x} in the argument of the distribution functions)

$$\hat{S}t(\mathbf{p}_1, \mathbf{p}_2, \mathbf{x}) = \frac{\pi g^2}{\hbar} \left[\delta(\hat{\Delta}(\mathbf{p}_1, \mathbf{p}_2; \mathbf{x})) ; \hat{\Upsilon}(\mathbf{p}_1, \mathbf{p}_2; \mathbf{x}) \right]_+ ; \quad \hat{\Upsilon} = N(\mathbf{p}_1 + \mathbf{p}_2) \left[1 - \hat{n}(\mathbf{p}_1) - \overline{\hat{n}(\mathbf{p}_2)} \right] - \hat{n}(\mathbf{p}_1) \overline{\hat{n}(\mathbf{p}_2)}. \quad (10)$$

The QKE scheme (7)–(10) is justifiable as long as the temporal [spatial] variation of the distribution functions is smooth on quantum scale, \hbar/T [$\hbar/\sqrt{mT\max(1, \sqrt{\epsilon_F/T})}$] [24]. Under these conditions, the expressions for the collision integral and spectral renormalization are local in time and space [26].

For densities of fermions, $\rho_a(\mathbf{x})$, bosons $\rho_b(\mathbf{x})$, and spin, $\mathbf{S}(\mathbf{x})$, we have

$$[\rho_a, 2\mathbf{S}, \rho_b](\mathbf{x}) = \int \frac{d\mathbf{p}}{(2\pi\hbar)^3} [\text{Tr} \hat{n}, \text{Tr} \hat{\sigma} \hat{n}, N](\mathbf{X}). \quad (11)$$

Quantum Kinetic Equations (8) respect the symmetries of model (1): conservation of mass, energy, spin rotation invariance, and when $\nabla U_{a,b} = 0$, Galilean invariance. The latter is implemented by

$$\begin{aligned} \hat{n}(\mathbf{p}, \mathbf{r}) &\rightarrow \hat{n}(\mathbf{p} - m\mathbf{v}, \mathbf{r} - \mathbf{v}t); \\ N(\mathbf{p}, \mathbf{r}) &\rightarrow N(\mathbf{p} - 2m\mathbf{v}, \mathbf{r} - \mathbf{v}t); \\ \hat{\mathcal{E}}(\mathbf{p}, \mathbf{r}) &\rightarrow \hat{\mathcal{E}}(\mathbf{p} - m\mathbf{v}, \mathbf{r} - \mathbf{v}t) + \mathbf{v} \cdot \mathbf{p} - m\mathbf{v}^2/2; \\ \Omega(\mathbf{p}, \mathbf{r}) &\rightarrow \Omega(\mathbf{p} - 2m\mathbf{v}, \mathbf{r} - \mathbf{v}t) + \mathbf{v} \cdot \mathbf{p} - m\mathbf{v}^2. \end{aligned} \quad (12)$$

Furthermore, for $\partial_t U_{a,b} = 0$ the equilibrium distributions

$$\hat{n} = \mathcal{F}\left(\frac{\hat{\mathcal{E}}(\mathbf{X}) - \mu - \hat{h}}{T}\right), \quad N = \mathcal{B}\left(\frac{\Omega(\mathbf{X}) - 2\mu}{T}\right), \quad (13)$$

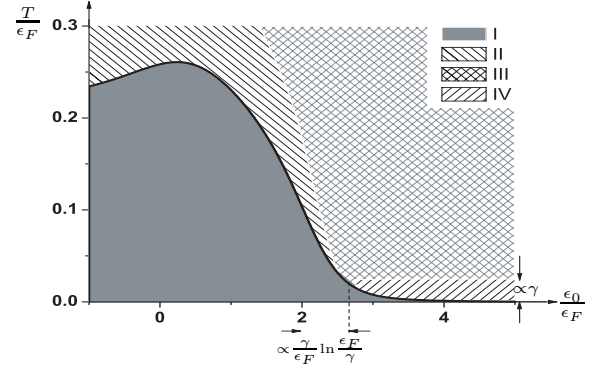


FIG. 2: Relevant regions of parameter space. Region I is the superfluid phase. The QKE (8b) is valid in II and III. Our results are valid above and close to the critical line (black curve). In III and IV the MT correction, Eq. (16), dominates at all times, whereas in II the bosonic dip, Eq. (17), dominates for $t < t_*$.

where $\mathcal{F}(x) = 1/(e^x + 1)$; $\mathcal{B}(x) = 1/(e^x - 1)$, solve Eqs. (8b), with arbitrary constants, μ, T , and an arbitrary traceless Hermitian 2×2 matrix, \hat{h} . The latter describes a possible stationary imbalanced spin population, which will be discussed elsewhere.

Transition temperature, T_c , is given by [27–31]

$$\epsilon_0 - 2\mu = \int_{-\frac{\mu}{2T_c}}^{\infty} \frac{dx}{\pi} \gamma(2\mu + 4T_c x) \left[\frac{\tanh x}{x} - \frac{1}{x + \frac{\mu}{2T_c}} \right]. \quad (14)$$

In the BCS regime, $\epsilon_0 - 2\epsilon_F \gg \gamma(\epsilon_F)$, this yields $T_c \propto \epsilon_F e^{-\frac{\pi}{2} \frac{\epsilon_0 - 2\epsilon_F}{\gamma(2\epsilon_F)}}$, and in the BEC regime, $(\epsilon_0 \lesssim 0)$, $T_c \approx T_{BEC} \simeq 0.218\epsilon_F$. The equilibrium bosonic spectrum, in the interval $T_c Gi < \Omega(\mathbf{q}) - 2\mu \lesssim T$ [25], is $\Omega(\mathbf{q}) = 2\mu + c + b\mathbf{q}^2$, with

$$c = \frac{2\gamma(E_1)}{\pi} \tau; \quad b = \frac{1}{4m} \left[1 + \frac{7\zeta(3)}{6\pi^3} \frac{\gamma(E_2)E_2}{T_c^2} \right]. \quad (15)$$

Here, $\zeta(3) \simeq 1.202$ (Riemann ζ -function), and

$$\left[\frac{2E_1^{1/2}}{\pi^2} E_2^{3/2} \right] = \int_{-\frac{\mu}{2T_c}}^{\infty} dx \left[\frac{\frac{1}{\cosh^2 x} (2\mu + 4xT_c)^{1/2}}{\tanh x - x \text{sech}^2 x} (2\mu + 4xT_c)^{3/2} \right].$$

In the BCS limit $T_c \ll \mu$, $E_{1,2} \rightarrow 2\mu \approx 2\epsilon_F$.

We now consider the evolution of a small density disturbance as shown in Fig. 1. A realistic experimental procedure [16] for creating such a disturbance is by applying a potential $U_a(\mathbf{x}) > 0$, much narrower than the size of the trap, for a time long enough for the system to equilibrate, with $n(\mathbf{X}, t = 0) = \mathcal{F}\left(\frac{\mathcal{E}(\mathbf{p}) - \mu + U_a(\mathbf{x})}{T}\right)$. At time $t = 0$, the potential is removed abruptly, leaving a small density disturbance. Next, we linearize the QKE and the initial distributions in the deviation from equilibrium. We solve the linearized fermionic kinetic equation (8a) perturbatively in the collision integral, Eq. (10), but treat the equilibrium decay and spectral renormalization of the bosons non-perturbatively,

consistent with our one-loop approximation. To zeroth order, the nonequilibrium part of the fermionic distribution evolves as that of free particles: $\delta n^{(0)}(\mathbf{X}, t) = \delta n(\mathbf{p}, \mathbf{x} - \mathbf{p}t/m, t = 0)$, giving rise to an expanding spherical density wave. Similarly for bosons, except for their exponential decay: $\delta N^{(0)}(\mathbf{X}, t) = \delta N(\mathbf{q}, \mathbf{x} - 2b\mathbf{q}t, t = 0) \exp\left[-\frac{(c+bq^2)\gamma(2\mu)}{2T_c}t\right]$. Deep in the BCS regime (regions III and IV in Fig. 2) the nonequilibrium bosons decay fast and can be neglected. Iterating the fermionic QKE, we find that in this regime, the correction to the density close to the origin is dominated by the same terms in the collision integral (10) that give rise to the MT correction to the conductivity of conventional superconductors. The latter can also be understood as the effect of Andreev reflection of electrons by superconducting fluctuations. For times $t \gg t_F \equiv \frac{\hbar\mu}{T_c^2\sqrt{\tau}}$, the density peak normalized by the total number of initially removed fermions is described (for $r \ll t\sqrt{\frac{2\mu}{m}}$) by

$$\frac{\delta\rho_F^{(1)}(r, t)}{|\delta\mathcal{N}_F|} \simeq \frac{\gamma(2\mu) \arctan\left(\frac{T_c^2}{\mu} \sqrt{\frac{\pi b}{\gamma(E_1)}} \frac{mr}{\hbar} \frac{t_F}{t}\right)}{8\hbar\mu\tau}. \quad (16)$$

In the BCS limit, this result is valid even for a broad resonance, i.e. not restricted by the condition Eq. (4).

In contrast to the exponential decay of any nonequilibrium deviation in the bosonic distribution, this sharp peak in the fermionic density decays only algebraically (one can show that this conclusion is not an artifact of perturbation theory), so that for large times the latter always dominates. For times smaller than $t^* = \frac{\pi\hbar}{\tau} \frac{T_c}{\gamma(2\mu)\gamma(E_1)}$, and close enough to T_c , the increasingly slow moving nonequilibrium bosons removed by the external potential [32] produce a density dip at the origin,

$$\frac{\delta\rho_B^{(0)}(r, t)}{|\delta\mathcal{N}_B|} \simeq -\frac{v\sqrt{\tau}}{\pi^2} e^{-\frac{\pi\gamma(2\mu)}{8bT_c\tau}[r^2 + \tau v^2 t^2]}, \quad (17)$$

where $v^2 \equiv 8b\gamma(E_1)/\pi$. This formula is valid for $t \gg t_B \equiv \hbar/\sqrt{\gamma(E_1)T_c\tau}$ and $r \ll t\sqrt{bT_c}$, and dominates the fermionic peak Eq. (16), reversing the sign of the density deviation at origin (see Fig. 1b). However, in the BCS regime, t^* is suppressed, and the peak described by formula (16) dominates at all times. The region where MT correction, Eq. (16), dominates is sketched in Fig. 2.

The critical behavior that was found in the response of the system to an external force may be of valuable use in current experiments on UCAG. It can provide for instance a new experimental tool to determine the transition temperature in the BCS side (where the formation of vortices [12] is not a strong enough mechanism), by applying a small density disturbance on the system and measuring nondestructively its subsequent evolution.

In summary, we undertook the study of the real time density evolution of a composite fermion-boson system in the BEC-BCS crossover, close to the transition point in

the normal phase. We found that in the far BCS regime a localized depletion of density will evolve into a sharp diverging peak where it was created, as a manifestation of critical fluctuations of the superfluid condensate. This phenomenon can be exploited in the experimental study of the crossover in fermionic UCAG's.

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